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The Treatment of Dielectrics in FasterCap

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The Treatment of Dielectrics in FasterCap

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***Abstract* – FasterCap and FastCap2 are quasistatic capacitance solvers able to handle arbitrarily shaped conductors embedded in multiple, piecewise-constant, dielectric regions. This paper reviews the underlying theory in FasterCap's treatment of dielectric regions, where the capacitance problem is re-formulated as an equivalent free-space one, where both the conductor-dielectric and the dielectric-dielectric interfaces are replaced by a surface-charge density layer, and the conductors and dielectrics are replaced by free space. Then a free-space Green's function is used to calculate the electric potential, considering the field generated by a total charge on the conductor-dielectric interfaces, and a polarization charge on the dielectric-dielectric interfaces. The relation between the source charges and the potential ultimately yields the required capacitance values.**

I. INTRODUCTION

The objective of this paper is to explain how the capacitance solvers FasterCap and FastCap2 can deal with arbitrarily shaped conductors embedded in multiple, piecewise-constant, dielectric regions. The theoretical ground for the employed method is well established, and the formal mathematical treatment can be found in [1] and [2] for the 3D case and the 2D case respectively.

In this paper we present a more intuitive and engineering view of this concept, applied to the 3D case. The 2D case requires a slightly different mathematical formulation, however similar considerations apply, and therefore it will not be explicitly presented in the paper.

Section II briefly reviews the mathematical formulation of the capacitance problem. In section III the solution method used in FasterCap and FastCap2 is described. Section IV demonstrates the method with some example cases.

II. MATHEMATICAL FORMULATION OF THE CAPACITANCE PROBLEM

As per definition, the capacitance is the ratio of charge to potential on an electrically charged, isolated conductor:

$$C=\frac{Q}{V} \tag{1}$$

Extending this definition to a set of m conductors, we can express this relation in the form of a Maxwell capacitance matrix [8]:

$$\begin{bmatrix} C_{1,1} & \cdots & C_{1,m} \\ \vdots & \vdots & \vdots \\ C_{m,1} & \cdots & C_{m,m} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_m \end{bmatrix} = \begin{bmatrix} Q_1 \\ \vdots \\ Q_m \end{bmatrix} \tag{2}$$

where the Maxwell capacitance matrix **C**, which is square, symmetrical, has real elements and dimension m, summarizes the relationship between conductor potentials V_i and conductor induced free charges Q_i. The positive diagonal element $C_{i,i}$ represents the self-capacitance of conductor *i*, while the negative off-diagonal element $C_{i,j}$ represents the capacitive coupling between conductors *i* and *j*.

Our goal is to calculate the elements of the capacitance matrix **C**. Recalling Coulomb's law, that gives a simple formula for the electric field of a stationary point charge in free space, we can readily derive the well known formula for the corresponding electric potential:

$$V(r)=\frac{q}{4\pi\varepsilon_0 r} \tag{3}$$

(plus an arbitrary constant V_o that we assume zero for the time being). In (3) q is the value of the charge, ε_o is the vacuum permittivity, and r is the distance of the charge from the observation point.

It is tempting therefore to try exploiting a similar relation, extended to an integration region over a set of charges, for directly calculating, one by one, the elements $P_{i,j}$ of a matrix **P**, that relates the charges **Q** to the voltages **V**,

$$\begin{bmatrix} P_{1,1} & \cdots & P_{1,m} \\ \vdots & \vdots & \vdots \\ P_{m,1} & \cdots & P_{m,m} \end{bmatrix} \begin{bmatrix} Q_1 \\ \vdots \\ Q_m \end{bmatrix} = \begin{bmatrix} V_1 \\ \vdots \\ V_m \end{bmatrix} \tag{4}$$

and invert this matrix to get the matrix **C**[1].

[1] The reader may wonder if there is a method to calculate the $C_{i,j}$ elements without passing through **P**. After all, Gauss's law gives a direct relationship between the charge and the electric field, which is the gradient of the potential. However, to apply Gauss's law, we must calculate the electric field, which is a function of the potentials applied on the conductors. So we need to solve first for the field generated in the space by a specific set of voltages **V**; and then calculate the charges on the conductors via Gauss's law. This is possible, and is the basis for the so-called indirect methods. They are called 'indirect' since there are two different solution steps involved. These are not,

With this goal in mind, let's formalize the required steps, starting with the Maxwell equations:

$$\nabla x \vec{E} = -\frac{\vartheta \vec{B}}{\vartheta t} \quad (5)$$

$$\nabla x \vec{H} = \frac{\vartheta \vec{D}}{\vartheta t} + \vec{J} \quad (6)$$

$$\nabla \cdot \vec{D} = \rho \quad (7)$$

$$\nabla \cdot \vec{B} = 0 \quad (8)$$

where $\vec{E}$ is the electric field strength, $\vec{H}$ is the magnetic field strength, $\vec{D}$ is the electric flux, $\vec{B}$ is the magnetic flux, $\vec{J}$ is the current density, and ρ is the free volume charge density. In the static case, the field quantities do not depend on time, therefore the magnetic and electric field become decoupled, being $\vartheta \vec{B}/\vartheta t$ and $\vartheta \vec{D}/\vartheta t$ null. For our electrostatic problem, we are interested in the electric field. Moreover, since the electrostatic field exists only outside the conductors (if the field was not null inside the conductors, the free charges would move, therefore the field would not be static), the current $\vec{J}$ is null: no conduction current and no impressed current. We are left therefore with the fundamental equations of the electrostatics[2]:

$$\nabla x \vec{E} = 0 \quad (9)$$

$$\nabla \cdot \vec{D} = \rho \quad (10)$$

In regions of space filled with homogeneous, isotropic, nondispersive, linear materials, also the following relation holds

$$\vec{D} = \varepsilon(r)\vec{E} \quad (11)$$

where ε(r) is the material permittivity, assumed to be constant in each of the regions of interest (i.e. piecewise constant).

A standard way to solve the system composed by (9), (10) and (11) is to leverage (9) to describe $\vec{E}$ by a scalar potential φ:

$$\vec{E} = -\nabla \phi \quad (12)$$

(since the curl of a gradient is always identically zero, (9) is automatically true), and upon substitution into (10) via (11) get the Possion's equation:

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon(r)} \quad (13)$$

To be able to solve this equation, we need to specify appropriate boundary conditions, matching our problem. We consider therefore, as boundary, the surface of the conductors S_c; the interfaces between regions filled with different dielectric materials S_d; and the potential at infinity. Note also that in the space $R^3 \setminus S$ between conductors and dielectric interfaces (that is, the total volume minus $S = S_c \cup S_d$) there are no free charges, so ρ is zero, and (13) simplifies to the Laplace's equation (14).

We have therefore the following boundary problem [6]:

$$\nabla^2 \phi = 0 \ , \quad \text{in } R^3 \setminus S \quad (14)$$

$$\phi = v \ , \quad \text{on } S_c \quad (15)$$

$$\phi^+ = \phi^- \ , \quad \text{on } S_d \quad (16)$$

$$\varepsilon^+ \frac{\vartheta \phi^+}{\vartheta n} = \varepsilon^- \frac{\vartheta \phi^-}{\vartheta n} \ , \quad \text{on } S_d \quad (17)$$

$$\lim_{|x| \to \infty} \phi(x) = 0 \quad (18)$$

where (15)-(18) provides the conditions on all the boundaries around the space of interest. In particular, (15) states that the potential on the surface of the conductors must be equal to the given potential v; (16) and (17) give the conditions across a dielectric interface, i.e. the potential must be a continuous function (16), and the normal component of the electric flux $\vec{D}_n$ must be preserved, n being the normal to the surface S_d (17); and the limit in (18) fixes zero as the value of the potential at infinity.

III. EQUIVALENT CHARGE FORMULATION

For capacitance calculation, only the induced free charge on the conductors is required. Remembering the idea given by (3), we can think to solve an equivalent problem, replacing the conductors and the dielectrics with free space, and account for their former influence on the potential field by placing a thin layer of stationary charges on S_d and S_c. We then calculate the potential field generated by this charge arrangement.

anyway, the methods used by FasterCap and FastCap2, which are direct. For a broader introduction to the topic, see [4].

[2] Note that these equations are valid also in the quasistatic case, when the wavelength of the frequency variation of the fields is large with respect to the dimensions of the structures of interest. A detailed formal derivation of this condition can be found in [7].

This approach is known in literature as the Equivalent-Charge Formulation (ECF).
In particular, since on each conductor-to-dielectric interface the total charge is the sum of free charge and polarization charge, we replace the conductor-to-dielectric interface with a surface charge density σ_c equivalent to the total charge density. This is shown in Fig. 1. Please note that stating that $\sigma_t = \sigma_f + \sigma_p$, where σ_t is the total charge density, σ_f is the free charge density, and σ_p is the polarization charge density, means considering σ_t, σ_f, σ_p as signed quantities. In general, σ_t will be, in absolute value, smaller than σ_f, since the free charge is compensated to some extent by the bounded dipole charges of the dielectric, that are aligned generating a net charge (due to the electric field). Note that the effect of the dielectric in contact with the surface of a conductor is accounted for with a relative permittivity value ε, in the relation $\sigma_f = \varepsilon\, \sigma_t$, i.e. the free conductor charge is ε times the total charge on the surface. Basically, the presence of the dielectric forces the accumulation of a charge ε times greater than the charge we would have without the dielectric, for the same applied potential difference. This is of course why high permittivity dielectrics are used as interposers between the plates of a capacitor to increase its capacitance value.

Fig. 1. Conductor-to-dielectric interface (a) and its Equivalent-Charge Formulation model (b)

In a similar way, since on each dielectric-to-dielectric interface the total charge is the polarization charge, we replace the dielectric-to-dielectric interface with a surface charge density σ_d equivalent to the total resulting polarization charge density. This is shown in Fig. 2. In this case, the different density of polarization charges at the interface gives a total polarization charge density.

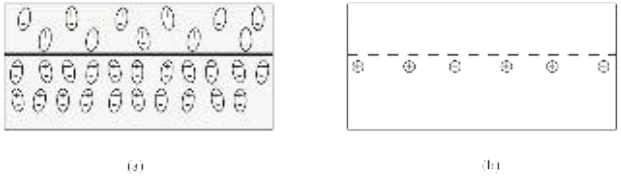

Fig. 2. Dielectric-to-dielectric interface (a) and its Equivalent-Charge Formulation model (b)

We have now an arrangement of charges in free space. To solve (14), we can therefore cast the differential equation into a boundary integral equation [7], writing the potential ϕ as superposition of the potentials due to the conductor surface charge density σ_c and the polarization surface charge density σ_d, as follows

$$\phi(x)=\int_{S_c}\frac{\sigma_c(x')}{4\pi\varepsilon_0|x-x'|}dS_{x'}+\int_{S_d}\frac{\sigma_d(x')}{4\pi\varepsilon_0|x-x'|}dS_{x'} \quad (19)$$

In (19), $x\in R^3$ is the point in space at with we evaluate the potential, due to conductor and polarization charges, and we made use of the so called Green's function for the free space [4]; more intuitively, however, (19) can be seen simply as an extension of (3).
We need now to meet the conditions (14)-(18). Note that the potential defined in (19) already satisfies the Laplace's equation (14), is continuous throughout R^3, satisfying (16), and decays like $1/x$, as requested by (18). We need therefore only to match the remaining boundary conditions (15) and (17). We therefore use (19), (15) and (17) to build a system of equations resembling (4), where we substitute φ as given by (19) into (17), and we solve for σ, given the potentials V_i on the conductors. Once the total charge densities σ_c are known, we readily get the free charges Q_i induced on the conductors, integrating the product of σ_c and ε of on the conductor areas:

$$Q_i=\int_{S_{ci}}\sigma_c(x)\varepsilon(x)dS_x \quad (20)$$

Note that if we neglected the multiplication by ε we would calculate only the total charges, but for the sake of capacitance calculation (2) we must consider the free induced charges, not the total ones.
We can now compute the terms C_{ij} of (2). This can be done in an easy way, defining a set of m unit voltage vectors $\mathbf{V_k}$ (i.e. the voltage vector $\mathbf{V_1}$ is composed by a first element V_1 is equal to one while the following elements are zero, the voltage vector $\mathbf{V_2}$ has the second element V_2 is equal to one and the other elements are zero, and so on). Therefore, multiplying by $\mathbf{V_1}$ in (2) will provide a vector $\mathbf{Q_1}$ that is composed by the elements of the first column of **C**. Solving in total m problems we completely find **C**. We use this method since our system of equations resembles (4), while there is no direct knowledge of (2); and the method described above is equivalent to inverting the matrix **P** in (4). It is worth adding that the reason why we don't invert **P** straight away is that the matrix inversion operation has a complexity of N^3, where N is the size of the matrix, becoming therefore very quickly computationally intractable as N grows. FasterCap and FastCap2, on the other hand, employ an acceleration scheme that allows

to treat the problem with a complexity of NM, where M is the number of conductors. This is because they can perform the matrix-vector multiplication **P·σ** in order N operations. The details about this acceleration method are not the goal of this article, and we'll not dig any further in the mechanism; for the interested reader, the information about FastCap2's algorithm can be found in [4] and [5].

A final note is due, for the sake of completeness. You will have noticed that the system of equations thus obtained via (19), (15) and (17), even if fully addressing the treatment of multiple, piecewise-constant dielectric regions in the capacitance calculation, is not really equivalent to (4). As a matter of fact, the system cannot be directly written as the product of a real matrix **P** by the vector **σ**, allowing a simple numerical solution. We'll sketch therefore the method used to transform these equations in a linear system that can be solved numerically. Let's assume that the charge distribution can be approximated by dividing the surface of the conductors in n panels, on each of which the charge density is constant. This is introducing an error, but if the number of panels is large enough (i.e. the discretization is fine enough), the error can be contained below a defined threshold. Thanks to the discretization, equation (19) can be rewritten as a sum, since on every panel the constant charge density σ can be taken out of the integral, as in (21)

$$\phi(x)=\sum_{k=1}^{nc}\sigma_{ck}\int_{S_{ck}}\frac{1}{4\pi\varepsilon_0|x-x'|}dS_{x'}+ \sum_{k=nc+1}^{n}\sigma_{dk}\int_{S_{dk}}\frac{1}{4\pi\varepsilon_0|x-x'|}dS_{x'} \qquad (21)$$

where nc is the number of panels into which the conductor surfaces have been divided, n-nc is the number of panels into which the dielectric interfaces have been divided, σ_{ck} is the charge density on the k-th conductor panel and S_{ck} is its surface, σ_{dk} is the surface charge density on the k-th dielectric panel, and S_{dk} is its surface. Let's now evaluate the potential φ(x) at the center of the first conductor panel, so that x is known[3]. The integrals now depends only on constants and geometrical parameters, that are fixed since the geometry does not change, and can be calculated, with standard integration methods, giving real numbers. Since the potential on the conductor's surface is one of our boundary conditions (15), we have, for the first panel

$$v_1=\sum_{k=1}^{nc}\sigma_{ck}\int_{S_{ck}}\frac{1}{4\pi\varepsilon_0|x_1-x'|}dS_{x'}+ \sum_{k=nc+1}^{n}\sigma_{dk}\int_{S_{dk}}\frac{1}{4\pi\varepsilon_0|x_1-x'|}dS_{x'} \qquad (22)$$

where x_1 is the center of the first panel, and v_1 is the potential on the first panel. Note that v_1 is known from **V**, since the whole surface of each conductor is at the same potential, and therefore the potential on each panel composing the conductor is the same. If we write other nc equations like (22), one for each panel of each conductor, we get a system of nc equations in n unknowns. Substitution of (21) into (17), which is the last boundary condition we did not use yet, allows to complete the problem statement, generating the remaining n-nc equations. One simple way to cast (17) with (21) in linear format is to approximate the derivatives in (17) using divided differences constructed near the target panel as in Fig. 3, yelding

$$\varepsilon^+\frac{\partial\phi^+}{\partial n}-\varepsilon^-\frac{\partial\phi^-}{\partial n}=0\approx\frac{(\phi_c-\phi^+)}{h}-\frac{(\phi^--\phi_c)}{h} \qquad (23)$$

and use (21) to evaluate the potential at the given points in space.

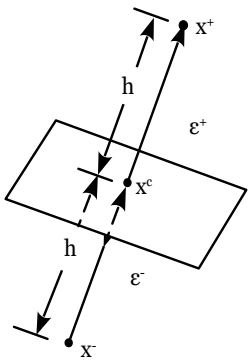

Fig. 3. The potential derivatives on both sides of a dielectric panel are approximated with divided differences

Writing n-nc equations, one for each panel into which the dielectric interfaces have been divided, provides therefore the last rows to complete the system of equations that can be summarized in matrix format as

[3] This scheme, that tests the value of the function at the centre of the panel, is called 'collocation'; other approaches are possible as well.

$$\begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \vdots & \vdots \\ p_{nc,1} & \cdots & p_{nc,n} \\ d_{nc+1,1} & \cdots & d_{nc+1,n} \\ \vdots & \vdots & \vdots \\ d_{n,1} & \cdots & d_{n,n} \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} v_1 \\ v_{nc} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (24)$$

where q_i is the total charge on panel i, v_i is the voltage on panel i, and $p_{i,j}$ and $d_{i,j}$ are the coefficients[4] coming from (22) and (23). Note that q_i is equal to σ_i / a_i, where a_i is the panel area, since we assumed that the surface charge density is constant over the panel; so we included the geometrical factor 1/ a_i directly in the coefficients $p_{i,j}$ and $d_{i,j}$.

The system (24) can be solved for a given set of potentials **v** to find the corresponding charges **q**. Please note that this system is not the same as (4), which consists of only m equations and considers the overall charges **Q** on the conductors and their potentials **V**. However (24) it is closely related to (4), since each conductor is ultimately composed by a set of panels. So, in free space, the sum of the charges from q_{c1} to q_{c2-1}, where c1 is the index of the first panel and c2-1 is the index of the last panel of the set of panels into which conductor 1 has been split, is equal to Q_1, and so on. On the other side, $v_{c1} = v_{c1+1} = \ldots = v_{c2-1} = V1$, since the surface of the conductors is equipotential. The final note is related to the link between **q** and **Q** when dielectrics are present. In this case, remember that we are using the Equivalent-Charge Formulation, so **q** is the total charge on the surface of the panels, not the free charge. We need therefore to account for the presence of the dielectric in contact with the surface of the conductor through (20), that leads to its discretized version, when considering discrete panels:

$$Q_i = \sum_{j=c_i}^{c_{i+1}-1} \varepsilon_j q_j \quad (25)$$

This equation is valid also if different parts of the conductor are in contact with different dielectric materials, allowing therefore a different relative permittivity ε_j for each panel, for treating cases as the one shown in Fig. 5.

IV. EXAMPLES

To illustrate the theory explained in the previous chapters, we'll now present two practical examples.

A. *Simple three-panels structure*

This first example is useful for exemplification purposes, but it is not intended to represent a real physical case. We will deal with a simple three-panel structure, as shown in figure, where the outmost panels are conductive panels embedded in different dielectric mediums, while the middle one represents the dielectric interface.

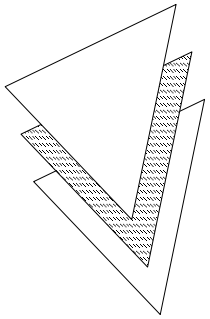

Fig. 4. Simple three-panels structure

There are two main approximations that make our problem not accurate with respect to the physical reality.

The first one is that, to keep the problem as simple as possible to demonstrate an application of the methods of the previous chapter, we'll assume that the charge density is constant all over the panels. This is of course far from true, because even in an approximate solution, we would split each panel in a set of smaller panels, to contain the charge density distribution error below a certain threshold.

The second one is that the dielectric interface is not complete, while in general it would encompass a closed region of space; we are therefore missing to force some of the boundary conditions.

Nevertheless, the method that we'll use for the solution is valid in general, as explained, and could be applied to a more complex structure yielding accurate results, as it will be shown for instance in the example B below.

We calculate now the geometrical integrals of (21) at the points x required to form the system (24) when $x = x_1$ and $x = x_2$, where x_1 and x_2 are the center of the first conductor panel and of the second conductor panel respectively, and as per the divided differences of (23) on the dielectric interface (with h small enough with respect to the panel dimensions). We obtain therefore the following numerical system (scaled by $4\pi\varepsilon_0$, i.e. the values must still be divided by $4\pi\varepsilon_0$):

[4] In literature, pi,j are often referred to as 'coefficients of potential'.

$$\begin{bmatrix} 37.699 & -2.569 & 2.569 \\ 1.703 & 4.814 & 0.951 \\ 1.703 & 0.951 & 4.814 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (26)$$

Panel number 1 is the dielectric interface, while panel number 2 and panel number 3 are the lower and upper conductor panels respectively. This can be seen also from the numerical values in (26), as follows. Elements $p_{2,2}$ and $p_{3,3}$ are the self coefficients of potential, i.e $p_{2,2}$ is the potential at the center of panels 2 due to the charge present on the same panel, and so on. Since the panels are geometrically identical, also the values are the same. These parameters are calculated from a term of an equation like (22), in the case of triangular panels:

$$p_{2,2} = \frac{1}{a_{T_1}} \int_{T_1} \frac{1}{|x_1 - x'|} dS_{x'} \quad (27)$$

The expression in (27) can be calculated evaluating the integral analytically or numerically, according to the required precision. Elements $p_{2,3}$ and $p_{3,2}$ are the mutual coefficients of potential, i.e. $p_{2,3}$ is the potential at the center of panel 2 due to the charge present on panel 3,; a similar observation is valid for $p_{3,2}$. Elements $p_{1,3}$, $p_{3,2}$, $p_{3,2}$ are relative to the dielectric panel interface, and are calculated through (23), evaluated at the center of the panel.

To obtain the capacitance we solve the system (26) for two **v** vectors: $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$ and $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. This is enough, since we only need to know the charge induced on the conductors when the first panel is raised to unit potential and the second is grounded, and vice-versa. Solving for $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$ and finally multiplying by $4\pi\varepsilon_0$ yields a charge vector q = $\begin{bmatrix} 0.196 & 2.347 & -0.533 \end{bmatrix}^T \cdot 10^{-11}$. This is therefore the charge arrangement that produces 1 volt on the first conductor panel and zero volt on the second conductor panel, and fulfils the boundary condition of continuity of the normal component of the electric flux **D** at the dielectric interface (of course within the used approximations). We remember however that the first conductor (the lower one) is embedded in a dielectric medium with relative permittivity equal to 2, so we must multiply the total charge by this factor to get the free induced charge on this conductor. The free charge values for the first and second conductor are therefore 46.94 pC and –0.533 pC respectively. These are the first two components of the Maxwell capacitance matrix **C** [8]. Solving for $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ and multiplying by $4\pi\varepsilon_0$ yields a charge vector q = $\begin{bmatrix} -0.196 & -0.417 & 2.463 \end{bmatrix}^T \cdot 10^{-11}$. Again multiplying by 2 the first charge value we get –0.834 pC and 24.63 pC. We have solved our problem and the resulting calculated capacitance matrix is therefore (in Farads):

$$\begin{bmatrix} 46.94 & -0.533 \\ -0.834 & 24.63 \end{bmatrix} \cdot 10^{-12} \quad (28)$$

For your reference, FasterCap's input file is reported below. The exact syntax is explained in [10]; for a basic understanding consider that each row begins with a declaration character. The asterisk '*' marks a comment; the 'C' directive indicates a file containing a geometrical description of a 3D structure in terms of triangular or quadrilateral panels; the 'D' directive specifies a dielectric interface; the 'T' directive specifies a triangular panel; the 'Q' directive specifies a quadrilateral panel. The comments interleaved in the input file should help you to understand the basic meaning of the definitions.

```
* Three plates structure
*
* lower contact
*
* triangle.txt : geometry file name
* 2.0 : permittivity
* 0.0 0.0 0.0 : 3D offset
C triangle.txt  2.0  0.0 0.0 0.0

* dielectric interface
*
* triangle.txt : geometry file name
* 2.0 : outer permittivity
* 1.0: inner permittivity
* 0.0 0.0 0.5 : 3D offset
* 0.25 0.25 0.0 : outer reference point
D  2.0 1.0  0.0 0.0 0.5  0.25 0.25 0.0

* upper contact
*
C triangle.txt  1.0  0.0 0.0 1.0

File triangle.txt

* simple triangle

* plate : conductor name
* 0 0 0  1 0 0  0 1 0 : 3D vertices
*                      of the triangle
T plate  0 0 0  1 0 0  0 1 0

End
```

B. Dielectric filled, finite plate thickness, parallel-plate capacitor

A real parallel-plate capacitor has armors with finite thickness. Fig. 5 shows the cross-section of such a capacitor, when the space between the plates is filled with a dielectric medium.

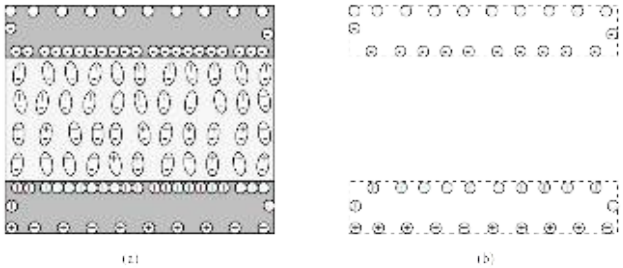

Fig. 5. Parallel plate capacitor with dielectric filler (a) and its Equivalent-Charge Formulation model (b). The side of each of the two conductive plates of the capacitor that are in contact with the dielectric material must accumulate more charges to counter-act the polarization charges with respect to the other three sides embedded in air.

In this case, the FasterCap input file specification must consider the fact that different sides of the conductors plates are in contact with different materials (i.e. air on the sides, the dielectric filler below or above). The FasterCap input file [10] is therefore:

```
* Capacitor
*
* This capacitor is made by a dielectric
* material with relative permittivity
* equal to 3.0, sandwiched between
* two square metal contacts
* with finite thickness

* lower contact
C capacitor_contact_sides.txt  1.0  0.0 0.0 0.0 +
C plate.txt  1.0  0.0 0.0 0.0 +
C plate.txt  3.0  0.0 0.0 0.2

* dielectric medium
D capacitor_diel_sides.txt  1.0  3.0  0.0 0.0 0.2
0.5 0.5 0.5 -

* upper contact
C capacitor_contact_sides.txt  1.0  0.0 0.0 0.8 +
C plate.txt  3.0  0.0 0.0 0.8 +
C plate.txt  1.0  0.0 0.0 1.0


File capacitor_contact_sides.txt
* conductor-dielectric interface
Q 1  1 1 0  1 0 0  1 0 0.2  1 1 0.2
Q 1  0 1 0  1 1 0  1 1 0.2  0 1 0.2
Q 1  1 0 0  0 0 0  0 0 0.2  1 0 0.2
Q 1  0 0 0  0 1 0  0 1 0.2  0 0 0.2
End

File capacitor_diel_sides.txt
* air-dielectric interface
Q 1  1 1 0  1 0 0  1 0 0.6  1 1 0.6
Q 1  0 1 0  1 1 0  1 1 0.6  0 1 0.6
Q 1  1 0 0  0 0 0  0 0 0.6  1 0 0.6
Q 1  0 0 0  0 1 0  0 1 0.6  0 0 0.6
End

File plate.txt
* top or bottom capacitor plate
Q 1  0 0 0  1 0 0  1 1 0  0 1 0
End
```

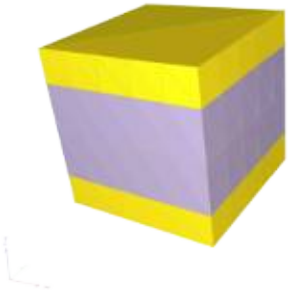

Fig. 6. Thick parallel plate capacitor 3d view.

V. CONCLUSION

This paper reviewed the theory behind FasterCap's and FastCap2's ability to handle the capacitance extraction of arbitrarily shaped conductors embedded in multiple, piecewise-constant, dielectric regions.

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□