The Maxwell Capacitance Matrix

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The Maxwell Capacitance Matrix

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Abstract – **The meaning of the Maxwell capacitance matrix is reviewed, highlighting the rationale of this matrix form in relation to the internal mechanisms of a capacitance field solver. Some practical examples also show how to use the Maxwell capacitance matrix to build a SPICE netlist.**

I. INTRODUCTION

The capacitance matrix calculated by FasterCap, FastCap2 and by most capacitance field solvers is in the form of a Maxwell capacitance matrix.
The Maxwell capacitance matrix topic is well explained in any good electromagnetic text book, but since this is a widely asked question, this paper will present the theory, with a practical approach.

II. RELATIONSHIP

A Maxwell capacitance matrix provides the relation between voltages on a set of conductors to charges on the conductors. For example, for a generic conductor set, the following relation is valid:

$$Q = C \cdot V \qquad (1)$$

where C is the Maxwell capacitance matrix, and V and Q are voltage and charge vectors respectively. The relation between the elements of C (Maxwell capacitance matrix) and the physical capacitances between a set of conductors will now be derived.
Consider for example a four conductors case, as shown in Fig. 1. Here the mutual and auto capacitances (capacitances towards infinity) are explicitly drawn.
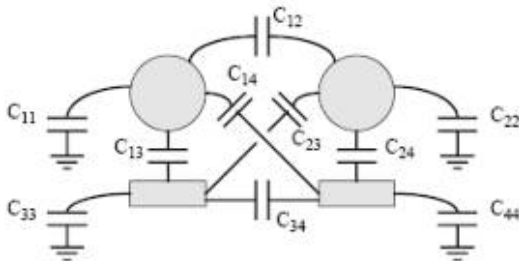

Fig. 1. Capacitances between four conductors

Physics tells us that the charge Q_1 on the conductor #1, given the voltages V_1, V_2, V_3, V_4 on the four conductors, is:

$$Q_1 = C_{11} \cdot V_1 + C_{12} \cdot (V_1 - V_2) + \\ + C_{13} \cdot (V_1 - V_3) + C_{14} \cdot (V_1 - V_4) \qquad (2)$$

which can be arranged as

$$Q_1 = (C_{11} + C_{12} + C_{13} + C_{14}) \cdot V_1 + \\ - C_{12} \cdot V_2 - C_{13} \cdot V_3 - C_{14} \cdot V_4 \qquad (3)$$

So the first row of the Maxwell capacitance matrix **C** is

$$|C_{11} + C_{12} + C_{13} + C_{14} \quad -C_{12} \quad -C_{13} \quad -C_{14}| \quad (4)$$

Completing the exercise, the full matrix has the form:

$$\begin{vmatrix} C_{11}+C_{12}+C_{13}+C_{14} & -C_{12} & -C_{13} & -C_{14} \\ -C_{21} & C_{21}+C_{22}+C_{23}+C_{24} & -C_{23} & -C_{24} \\ -C_{31} & -C_{32} & C_{31}+C_{32}+C_{33}+C_{34} & -C_{34} \\ -C_{41} & -C_{42} & -C_{43} & C_{41}+C_{42}+C_{43}+C_{44} \end{vmatrix} \quad (5)$$

Extending to the general case, the Maxwell capacitance matrix has the form:

$$\begin{vmatrix} C_{11}+C_{12}+\cdots+C_{1n} & -C_{12} & \cdots & -C_{1n} \\ -C_{21} & C_{21}+C_{22}+\cdots+C_{2n} & \cdots & -C_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ -C_{n1} & -C_{n2} & \cdots & C_{n1}+C_{n2}+\cdots+C_{nn} \end{vmatrix} \quad (6)$$

It is worth noting that the capacitance values to be used on a SPICE-like simulator are the C_{xy} values, and not directly the values appearing as elements

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in the Maxwell capacitance matrix. For instance, in our four conductor example, the element in the first column, first row of the Maxwell capacitance matrix is $C_{11}+C_{12}+C_{13}+C_{14}$, while in SPICE you should define a capacitor between node 1 and GND with the value C_{11}, a capacitor between node 1 and node 2 with the value C_{12}, etc. This means that some simple operations are in general needed to derive the C_{xy} values from the Maxwell capacitance matrix.
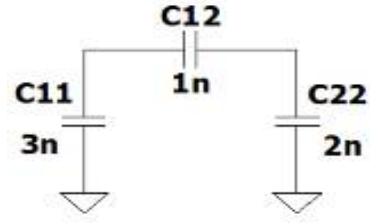

Fig. 2. Spice equivalent circuit

□

III. MOTIVATION

The reason why the capacitance field solvers, like FastCap2 and FasterCap, calculate the capacitance matrix in the form of a Maxwell capacitance matrix it that, since in general the relation (1) holds, to calculate the k-th column of **C**, you can set the potential of the k-th conductor to one (i.e. the k-th element of **V** is set to 1) and the potential of all remaining conductors to zero (i.e. all n-1 remaining elements of **V** are set to 0). The resulting **Q'** vector will be therefore equal to the k-th column of **C**.

In general, knowledge of the **C** matrix gives you all the elements needed to calculate the Q-V relation (i.e. the capacitance) for any set of voltages **V**.

IV. EXAMPLES

For instance, let's consider a Maxwell capacitance matrix for a two-conductors problem:

$$\begin{bmatrix} 4\,nF & -1\,nF \\ -1\,nF & 3\,nF \end{bmatrix} \qquad (7)$$

From this matrix, we can easily find that:

$$C_{11}=4\,nF-1\,nF=3\,nF$$
$$C_{22}=3\,nF-1\,nF=2\,nF \qquad (8)$$
$$C_{12}=C_{21}=-(-1\,nF)=1\,nF$$

The corresponding SPICE netlist will then resemble the following text:

```
C11   node1 0      3n
C22   node2 0      2n
C12   node1 node2  1n
```

and is represented in Fig. 2.

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